HEAT EXCHANGERS

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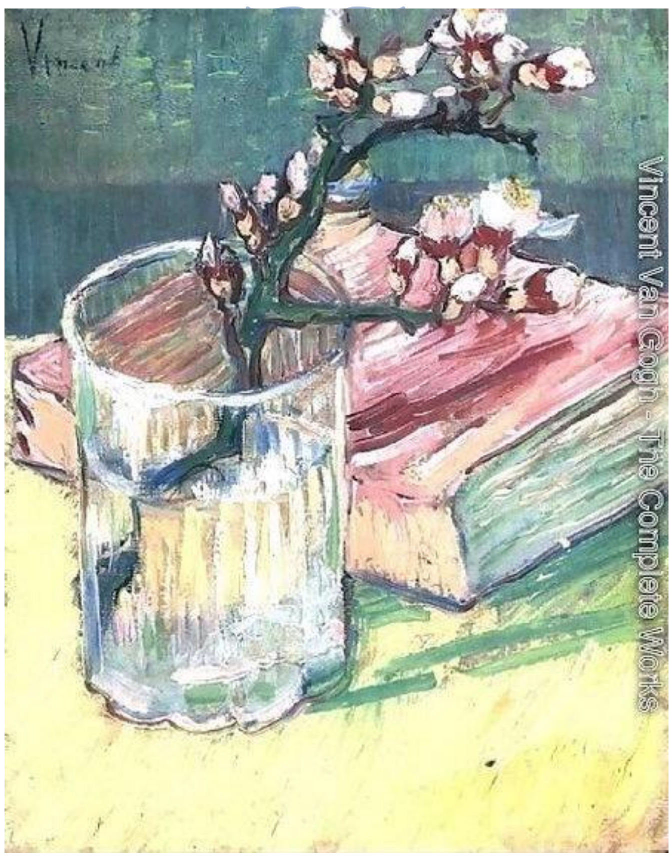

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Abstract

The goal of this paper is to quantitatively characterize the overall heat transfer coefficient times the area of heat transfer (UA) for a series of heat exchangers using multiple analytical methods, and to analyze the effects of hot and cold water flow rates on UA. Five heat exchanger configurations were tested in this paper. The first two were double pipe heat exchangers. One was set in a co-current configuration and the other was set in a counter-current configuration. UA was calculated using three methods: the first from an energy balance on the hot stream, the second from an energy balance on the cold stream, and the third using empirical correlations. The values for UA calculated by these methods ranged from around $2\ to\ 35 \frac{W}{m^2K}$ at varying flow rates of hot and cold streams. Each method predicted different UA values, with the energy balance on the cold stream predicting the highest, the empirical correlations predicting the lowest, and the energy balance on the hot stream predicting values somewhere in the middle. In addition, three shell and tube heat exchangers were tested. Two were single pass in co-current and counter-current configurations. The third was a Multipass (4-pass) configuration. The UA of each system was calculated using the Number of Transfer Units (NTU) method to return UA values that ranged from around $3\ to\ 130 \frac{W}{m^2K}$ at varying flow rates of hot and cold streams.

Introduction

Heat exchangers can offer an efficient means of facilitating heat transfer (HT) between a cold and a hot fluid. Heat exchangers can modify the rate of HT by utilizing high or low heat capacity fluids, high overall heat transfer coefficients, high surface areas, and by modifying the direction of flow.

$$q = \dot{m}C_{p,i}\Delta T$$
$$q = Rate\ of\ Heat\ Transfer\ (W)$$
$$\dot{m} = Mass\ Flow\ Rate\ \left(\frac{g}{s}\right)$$
$$C_{p,i} = Heat\ Capacity\ of\ Fluid\ i\ \left(\frac{J}{gK}\right)$$
$$\Delta T = Temperature\ Difference\ Between\ Inlet\ and\ Outlet\ of\ Hot\ or\ Cold\ Fluid\ (K)$$

Eq 1. Heat transfer rate to cold fluid or from hot fluid in a heat exchanger.[6]

The heat capacity of a fluid, C_p, is a property that is different for every type of fluid substance. For liquids, such as the water used in the experiment, heat capacity remains relatively constant as temperature changes with a value of around $4.18\frac{J}{gK}$ for water. Given a constant mass flow rate and rate of heat transfer, the temperature difference between the inlet and outlet of a liquid will be inversely proportional to the heat capacity of the liquid. Often, fluids with high heat capacities such as water are used since they can absorb large amounts of heat with minimal temperature changes.

$$q = UAF\Delta T_{lm}$$
$$U = Overall\ Heat\ Transfer\ Coefficient\ \left(\frac{W}{m^2K}\right)$$
$$A = Surface\ Area\ of\ Heat\ Transfer\ (m^2)$$
$$F = Correction\ Factor$$
$$\Delta T_{lm} = Logarithmic\ Mean\ Temperature\ Difference\ (K)$$

Eq 2. Heat transfer for an adiabatic heat exchanger.[6]

$$T_{lm} = \frac{\Delta T_i - \Delta T_o}{\ln\left(\frac{\Delta T_i}{\Delta T_o}\right)}$$

$\Delta T_i = Temperature\ Difference\ Between\ Hot\ and\ Cold\ Stream\ at\ Inlet\ (K)$

$\Delta T_o = Temperature\ Difference\ Between\ Hot\ and\ Cold\ Stream\ at\ Outlet\ (K)$

Eq 3. Logarithmic Mean Temperature Difference.[6]

The overall driving force of HT for heat exchangers can be described by eq 2., which was originally derived analytically for an adiabatic double pipe heat exchanger but has been modified to include other adiabatic heat exchanger types that are more difficult to characterize analytically. To do this, the correction factor, F, accounts for any differences between a double pipe heat exchanger and the heat exchanger configuration of interest. The value for F should range from $0 < F \leq 1$ for all heat exchangers.

The overall heat transfer coefficient is determined by the sum of thermal resistances in a system:

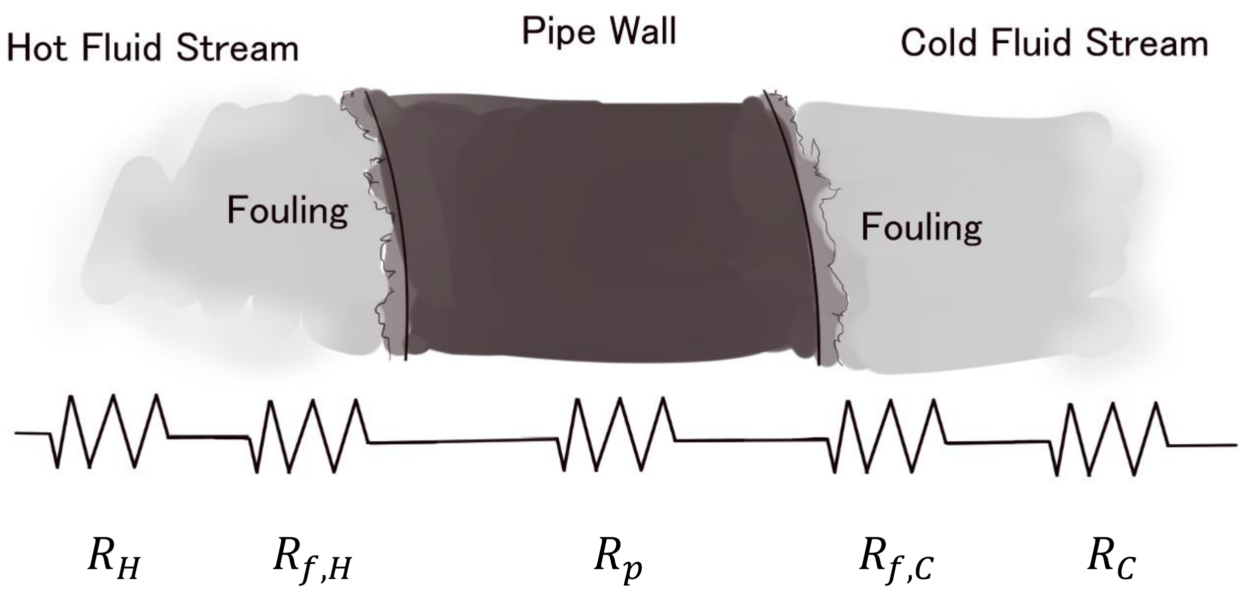

Figure 1. Thermal resistance analogy applied to pipe.

$U = \left(R_C + R_{f,C} + R_p + R_{f,H} + R_H\right)^{-1}$

$R_C = Resistance\ in\ Cold\ Stream\ due\ to\ Convective\ Heat\ Transfer$

$R_{f,C} = Resistance\ in\ Cold\ Stream\ due\ to\ Fouling$

$$R_p = Resistance\ of\ Pipe\ due\ to\ Conductive\ Heat\ Transfer$$

$$R_{f,H} = Resistance\ in\ Hot\ Stream\ due\ to\ Fouling$$

$$R_H = Resistance\ in\ Hot\ Stream\ due\ to\ Convective\ Heat\ Transfer$$

Eq 4. Overall Heat Transfer Coefficient. Units of resistances are the inverse of the units of the overall heat transfer coefficient.[6]

Heat exchangers are designed to keep each resistance low to maximize heat transfer. In most systems there is one resistance that is far higher than the others, meaning that the resistance is rate limiting to the amount of heat transfer that can occur in a heat exchanger. The resistance in the pipe, R_p, is minimized by increasing the conductivity of the pipe and decreasing the thickness of the pipe. The resistances due to fouling are due to the buildup of solid on the surface in which heat transfer occurs and increase with the lifetime operation of a heat exchanger. Fouling is difficult to control for since cleaning the inside of a heat exchanger is difficult due to their complex design. If the resistance due to fouling increases too much the heat exchanger may cease to function. The resistances of the cold and hot stream due to convective heat transfer are of particular interest because they can be controlled by varying the flow rate of the cold and hot fluids. The convective heat transfer resistances are inversely proportional to the individual convective heat transfer coefficient of the stream. This means that as flow rate increases, the convective heat transfer resistances should decrease. Identifying the limiting resistance in the system can help engineers decide ways in which to efficiently and cost effectively increase the overall heat transfer in a heat exchanger. While it would seem ideal to make flow rates infinitely high it is important to keep in mind that a fluid must remain in a system for certain amount of time for any noticeable change in temperature to occur. At very high flow rates a fluid may pass through a system without the desired amount of heat exchange occurring.

To achieve a desired heat transfer rate in a heat exchanger a surface area of the proper size is needed. High surface areas provide lots of contact points between the hot and cold streams for heat transfer to occur. Certain heat transfer equipment is built to maximize the surface area of heat transfer.

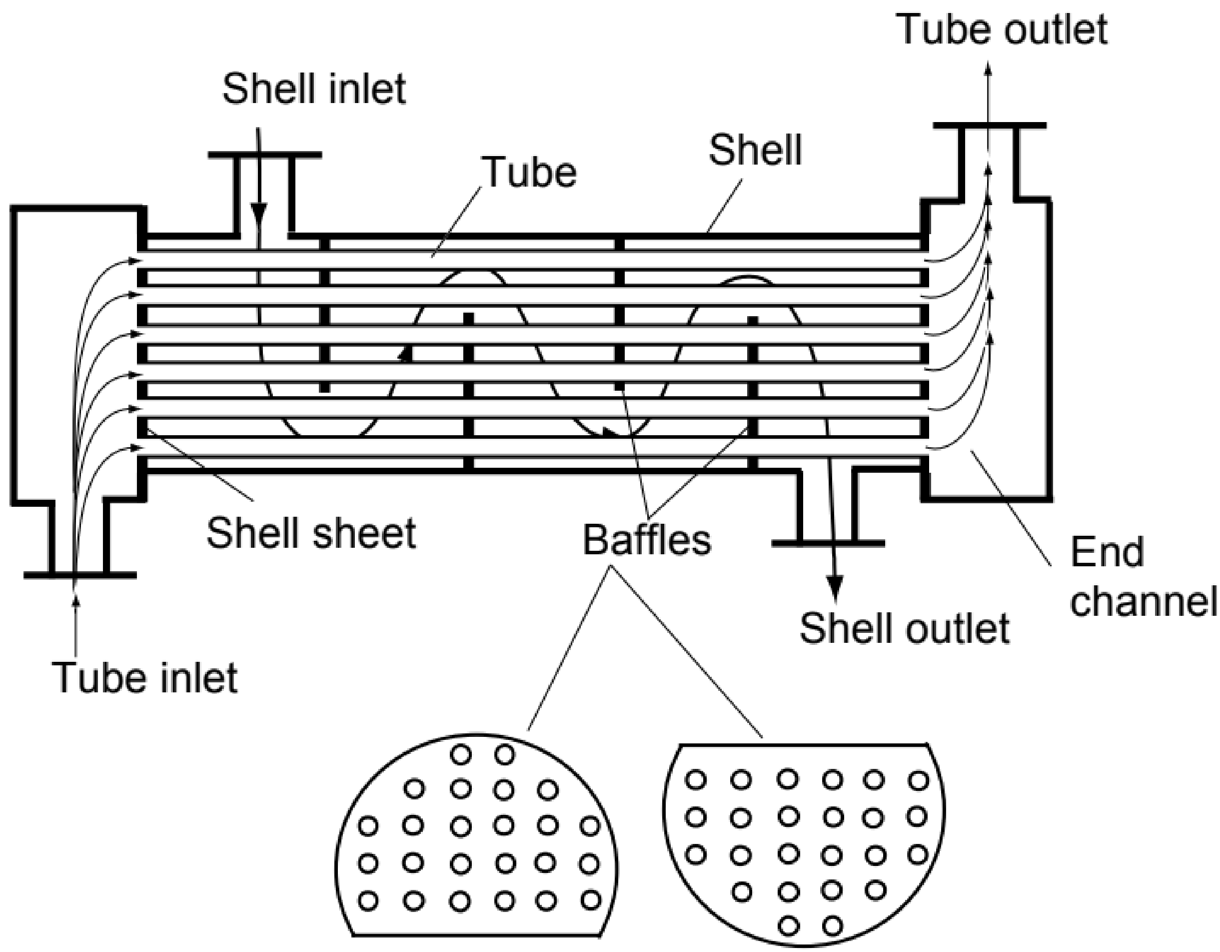

Figure 2. A typical single pass shell and tube heat exchanger.[6]

Shown in figure 2, shell and tube heat exchangers are built with the objective of maximizing the surface area between the hot and cold streams by splitting the flow of one of the streams into multiple tubes. The other stream passes through the shell, where it encounters baffles which prevents backflow and increases the amount of time the fluid spends in the heat exchanger so that the fluid in the shell makes contact with all the tubes, rather than just passing through the heat exchanger.

The tubes in a shell and tube heat exchangers may undergo one or multiple passes through a shell. In a multipass heat exchanger, the liquid in the tube, instead of leaving the system after one pass through the shell, is diverted through the shell three additional times. This gives systems with multiple passes (multipass heat exchangers) a higher surface than those with single passes. When compared to single pass shell and tube systems, multipass heat exchangers provide a higher surface area and require a longer time for the liquid to pass through from inlet to outlet. This means that multipass heat exchangers can provide higher amounts of heat exchange than single pass heat exchangers with comparable volumes.

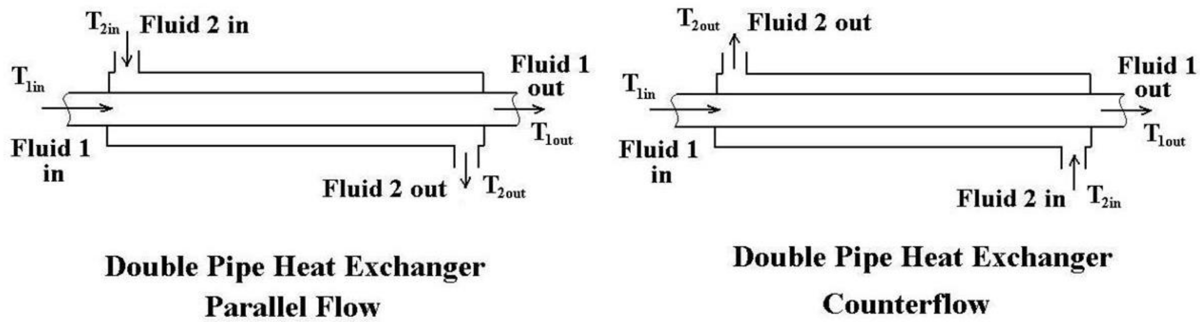

Figure 3. Double Pipe Heat Exchanger with co-current (parallel) flow and counter-current flow. [6]

Shown in figure 3, double pipe heat exchangers (sometimes referred to as single tube heat exchangers) provide less surface area, but are significantly cheaper to purchase (or build) when compared to shell and tube heat exchangers.

The direction of flow in a heat exchanger can either be co-current with the hot and cold streams flowing in the same direction or counter-current with the hot and cold streams flowing in opposite directions. As shown in figure 3, co-current flow in a double pipe heat exchanger can be achieved by placing the inlet of hot and cold streams on the same side of the heat exchanger. Counter-current flow can be achieved in a double pipe heat exchanger by placing the inlet of the cold and hot streams on opposite sides of the heat exchanger. The same is true for a single pass shell and tube heat exchanger. For multipass heat exchangers, since the direction of the tube flow changes with every pass, counter-current and co-current flow may both occur.

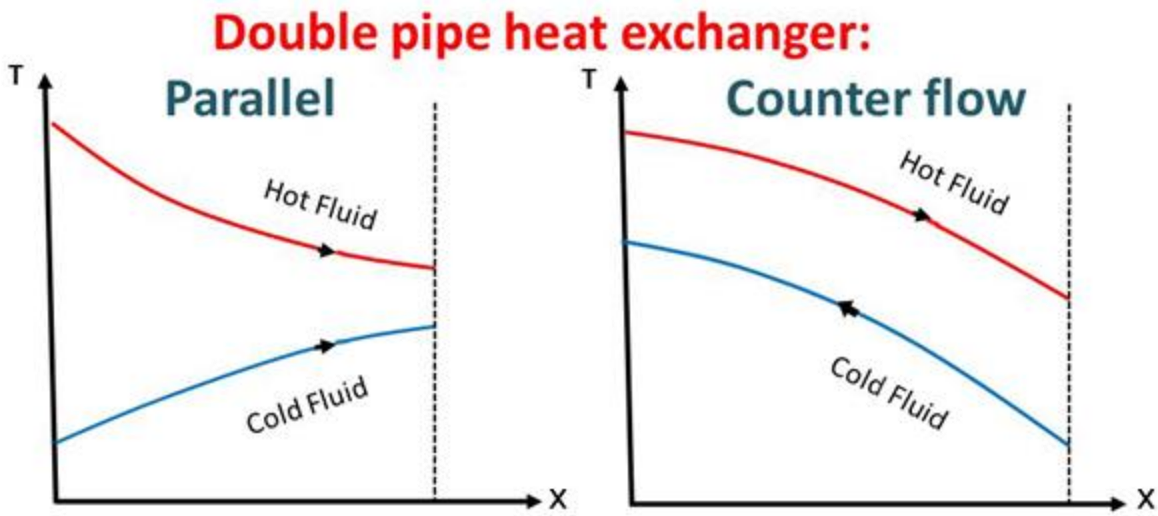

Fig 4. The temperature in the hot and cold streams down the length of a double pipe heat exchanger for co-current (parallel flow) and counter-current flow. Where T is temperature and x is position.

Depending on which flow configuration (co-current or counter-current flow) is used, the amount of heat transfer changes. Co-current provides a large temperature difference near the inlet of the heat exchanger, meaning that for very small heat exchangers co-current flow can provide a reasonable amount of heat transfer. At larger lengths, the temperatures of the hot and cold streams will equalize, meaning that no additional heat transfer can occur. The driving force, the difference in temperature between the cold and hot stream, will become zero. This is useful is a specific temperature is required of the streams coming out of a heat exchanger. When utilizing counter-current flow, the driving force usually remains non-zero down the length of the heat exchanger. Since the driving force is never depleted in counter-current flow, while it can become zero in co-current flow, counter-current flow provides higher rates of heat transfer than co-current flow.

Empirical correlations may be used to estimate the values of resistances within a heat exchange system. For the double pipe heat exchanger system, the following equations could be used to model the individual resistances of the system, given that the cold stream in the annulus (between the outer and inner pipe) and the hot stream is in the inside pipe. By knowing the value of each resistance, the overall heat transfer coefficient may be found using equation 4

Reynold's Number

$$Re_D = \frac{uD_e}{\nu}$$

Eq 5. Reynolds Number in Pipe Flow.[6]

For Inner Tube:

$$D_e = D_{i,i}$$

Eq 6 Hydraulic Diameter for Inner Tube.[6]

For Annulus:

$$D_e = D_{i,o} - D_{o,i}$$

Eq 7. Hydraulic Diameter for Annulus.[6]

Nusselt Number:

$$Nu_D = \frac{hD_e}{k_f}$$

Eq 8. Nusselt number for pipe flow.=6

For Inner Pipe Flow:

Laminar:

$$Nu_D = \frac{h_i D_e}{kf} = 1.86\left(\frac{D_{i,i} Re Pr}{L_p}\right)^{\frac{1}{3}}$$

Eq. 9 Correlation for Nusselt number for laminar flow in pipe.[1]

Turbulent:

$$Nu_D = \frac{\left(\frac{f}{2}\right)(Re_D - 1000)Pr}{1 + 12.7\left(\frac{f}{2}\right)^{\frac{1}{2}}\left(\mathrm{Pr}^{\frac{2}{3}} - 1\right)}$$

$$f = (1.58\ln(Re_D) - 3.28)^{-2}$$

Eq. 10 Correlation for Nusselt number for turbulent flow in pipe.[5]

For Annulus Flow:

$$Nu_D = \frac{h_a D_e}{kf} = 1.2\left(3.66 + \frac{0.0668Gz}{1 + (0.04Gz)^{\frac{2}{3}}}\right)$$

$$Gz = \frac{RePrD_e}{L_p}$$

Eq. 11 Correlation for Nusselt number for laminar flow in annulus. [5]

Thermal Resistance Calculations:

The individual heat transfer coefficients can be used to calculate the convective HT resistances described by equation 4.

$$R_H = \frac{1}{\pi h_i D_{i,i} L_p}$$

Eq. 12 Convective HT Resistance Pipe Side

$$R_C = \frac{1}{\pi h_a D_{o,i} L_p}$$

Eq. 13 Convective HT Resistance Annulus Side

The resistance due to the copper pipe can be calculated using the following formula:

$$R_p = \frac{\ln\left(\frac{D_{o,i}}{D_{i,i}}\right)}{2\pi k_{copper} L_p}$$

Eq. 14 Conductive HT Resistance in Pipe

Terminology:

$Nu_D = Nusselt\ Number\ (average)$

$h = Individual\ HT\ Coefficient\ \left(\frac{W}{m^2K}\right)$

$k_f = Fluid\ Conductive\ HT\ Coefficient\ \left(\frac{W}{mK}\right) = .597\frac{W}{mK}\ at\ 293\ K\ for\ water$

$Re_D = Reynold's\ Number\ (average)$
$u = Velocity\ of\ Stream\ \left(\frac{m}{s}\right)$
$D_e = Hydraulic\ Diameter\ (m)$

$\Delta T_{cold} = Temperature\ Difference\ Between\ Inlet\ and\ Outlet\ of\ Cold\ Stream\ (K)$
$h_i = Individual\ Heat\ Transfer\ Coefficient\ for\ Pipe\ \left(\frac{W}{m^2K}\right)$
$h_a = Individual\ Heat\ Transfer\ Coefficient\ for\ Annulus\ \left(\frac{W}{m^2K}\right)$
$L_p = Measured\ length\ of\ pipe\ (m)$
$D_{i,i} = Measured\ Inner\ Diameter\ of\ Inner\ Tube\ (m)$
$D_{i,o} = Measured\ Inner\ Diameter\ of\ Outer\ Tube\ (m)$
$D_{o,i} = Measured\ Outer\ Diameter\ of\ Inner\ Tube\ (m)$

Procedure

Setup and Materials

Double Pipe Heat Exchanger

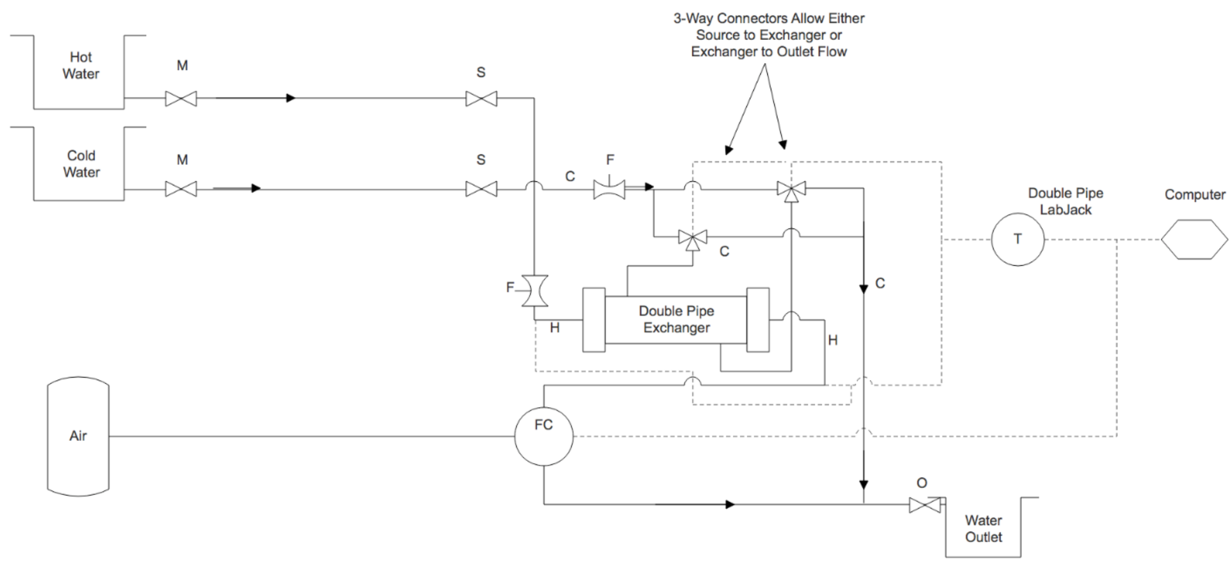

Fig 4. PID for Double Pipe Heat Exchanger. Heat exchanger can be configured in counter-current and co-current flows by diverting cold water in a three-way valve.

The double pipe heat exchanger was made from a small copper pipe placed within a larger copper pipe. Hot water flowed through the inner pipe and cold water flow through the annulus between the pipes. The outer pipe was uninsulated and exposed to room temperature air (around 24 ℃).

Inner Tube Inner Diameter = $14.39\ mm$

Inner Tube Outer Diameter = $15.82\ mm$

Outer Tube Inner Diameter = $26.60\ mm$

Outer Tube Outer Diameter = $28.48\ mm$

Length of Heat Exchanger = $0.9398\ m$

BCF Shell and Tube Heat Exchanger:

Co-current Flow Heat Exchanger

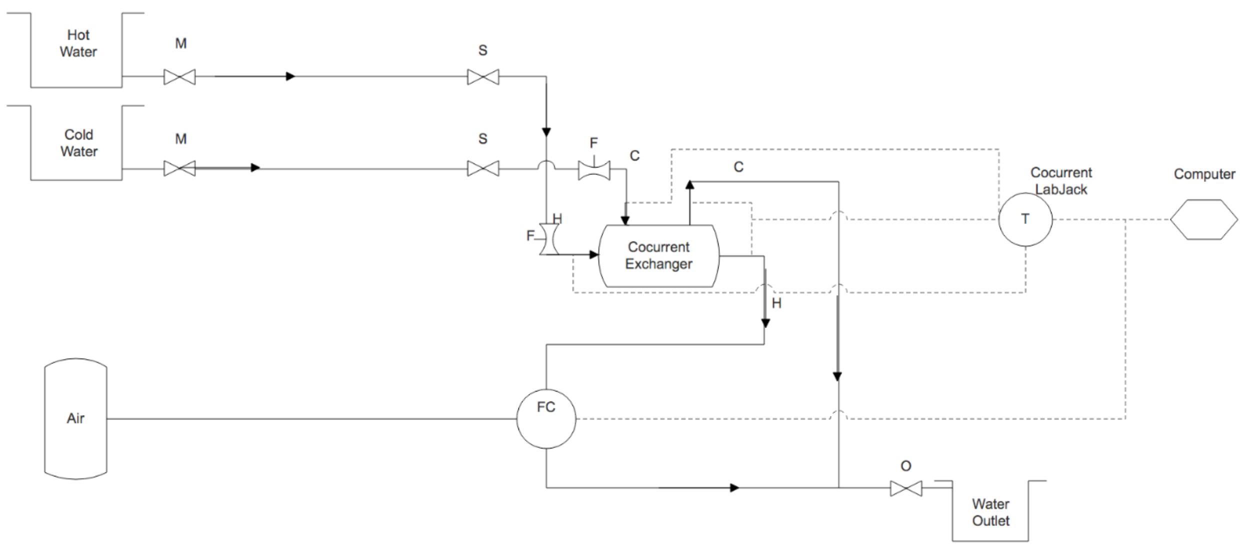

Figure 5. PID of co-current heat exchanger.

Counter-Current Heat Exchanger

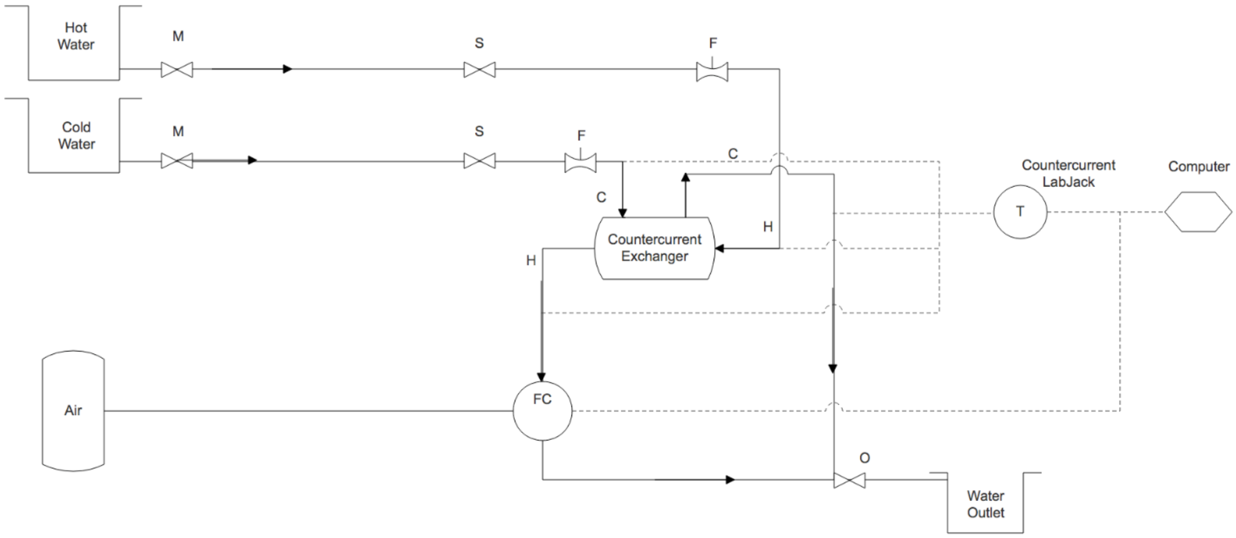

Figure 6. PID of counter-current heat exchanger.

The co-current and counter-current heat exchangers were made from the same model of heat exchangers (single pass shell and tube) with different flow configurations. For the co-current heat exchanger, hot and cold water entered the system on the same side of a shell and tube heat exchanger. For the counter-current heat exchanger, hot and cold water entered the system on opposite sides of the shell and tube heat exchanger. The tubes were made from a copper alloy and the shell was made from brass.

Multipass Heat Exchanger

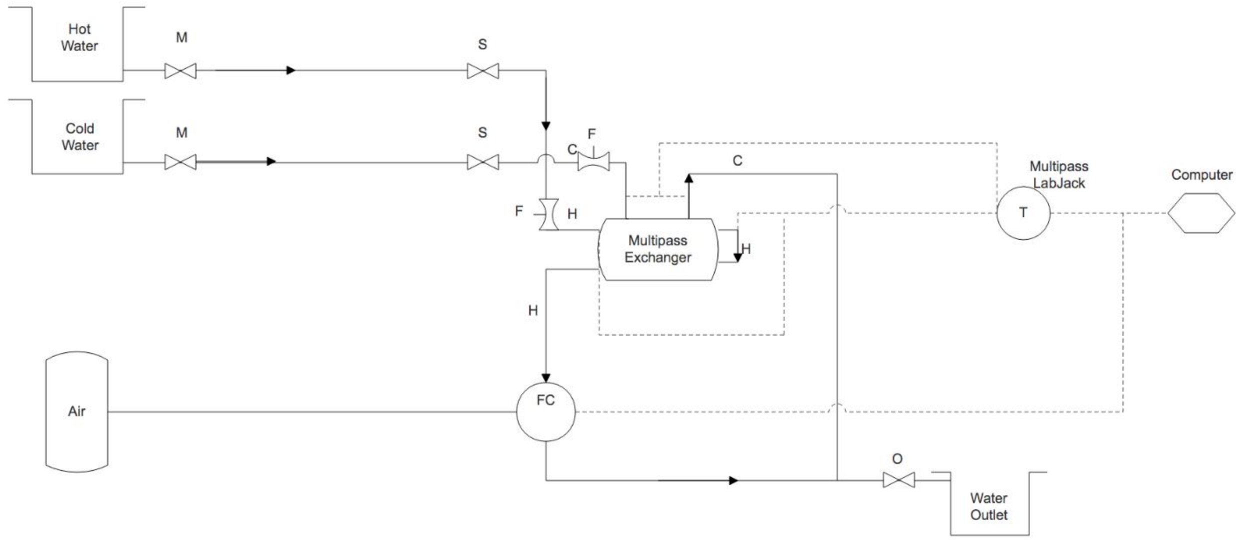

Figure 7. PID of multipass heat exchanger.

The multipass heat exchanger was a 4-pass shell and tube heat exchanger with the same volume and is made of the same materials as the co-current and counter-current shell and tube heat exchangers.

To operate each system, the air flow valve must first be opened so that the flow control valves (labeled FC) can be opened to max capacity. The hot and cold water main valves, labeled M, must also be opened. Then a LabVIEW program, connected to each heat exchanger through LabJacks, may be used to view and record the temperature measured by the temperature sensor chips (a TMP) placed at the inlets and outlets of each heat exchanger.

For any readings of temperature taken outside the heat exchanger systems, a type K thermocouple was used.

The diameter of each tube in the shell and tube heat exchangers was measured to be 5.5 mm.

Material Properties

Thermal Conductivity of Copper at 298 K $= k_{copper} = 386\,\frac{W}{mK}$

Thermal Conductivity of Brass at 298 K $= k_{brass} = 107\,\frac{W}{mK}$

Procedure

Because of the setup of the heat exchangers, the flow rate of cold and hot water to each heat exchanger could be changed simultaneously. At all times the flow rates of cold and hot water to each heat exchanger were kept nearly identical by opening the valves to each heat exchanger equal amounts. The flow rate passing through a given valve was labelled on the front panel of the valve and could be controlled manually by turning a knob at the bottom of the front panel. The flow rates are given in GPM, so they must be converted to metric in order to be used in conjunction with the metric temperature readings.

The following procedure was performed two times. The first time the double pipe was set to a co-current configuration and the second time the double pipe was set to a counter-current configuration.

The cold water flow rate was varied from 0.2 GPM (12.6 mL/s) to 0.8 GPM (50.5 mL/s), in increments of 0.2 GPM (12.6 mL/s). At each increment of the cold water flow rate the cold water flow rate was held constant, while the flow rate of hot water was varied stepwise from 0.2 GPM (12.6 mL/s) to 1.0 GPM (63.1 mL/s), in increments of 0.2 GPM (12.6 mL/s). Each time the cold or hot water flow rate was varied, the system was given time (around 3-10 minutes) to reach steady state. When around 50 seconds or more had passed at steady state, the cold or hot water flow rate was varied to the next incremental value. Data for temperature of the inlet and outlet of each stream was recorded using a LabVIEW program. Data was interpreted using Excel to sort data and Python for data analysis.

Methodology for Calculating UA

UA, the overall heat transfer coefficient times the heat transfer area was used to characterize each heat exchanger system. UA was used for characterization, rather than the overall heat transfer coefficient alone, because it provides a better understanding of the amount of heat transfer a heat exchanger can provide. In addition to convert UA to the overall heat transfer coefficient, it would have to be divided by the measured area of heat transfer for each heat exchanger. Measuring the surface area was simple for the double pipe heat exchanger, but measuring the surface area of heat transfer for each shell and tube configuration was difficult to perform accurately. While the company that produced each shell and tube lists that the area of heat transfer is 2.4 ft^2, fouling and deformations could affect the surface area of heat transfer and would be very difficult to account for without dissecting the entirety of each heat exchanger.

For the double pipe heat exchanger, to characterize UA three methods were used. Three methods were used because the system was non-adiabatic. The heat transfer to the cold stream was shown to be unequal to the heat transferred from the hot stream. The first two methods attempt to show the effect of applying an equation that assumes adiabatic heat transfer to a non-adiabatic system. To do this, the first two methods assume that the energy output or gained by a fluid in the system (equation 1) is equal to the energy transferred between the two streams (equation 2), i.e., equation 1 = equation 2. Method one utilized equation 1 with respect to the hot fluid and method two utilized equation 1 with respect to the hot fluid. Since a double pipe is being used the correction factor, $F, = 1$ in equation 2. The third method utilized empirical equations from literature based on the Reynolds Number of the inner pipe and annulus to calculate the overall heat transfer coefficient, which was multiplied by the measured area of the heat exchanger.

Method 1:

Energy Balance:

$$q = \dot{m}_{hot} C_{p,H_2O} \Delta T_{hot} = (UA) F \Delta T_{lm}$$

Solving for UA:

$$UA = \frac{\dot{m}_{hot} C_{p,H_2O} \Delta T_{hot}}{\Delta T_{lm}}$$

Eq. 15. Characterizing the UA of the double pipe heat exchanger using the hot fluid in the energy balance. For double pipe heat exchanger correction factor, F = *1.*

Method 2:

Energy Balance:

$$q = \dot{m}_{cold} C_{p,H_2O} \Delta T_{cold} = (UA) F \Delta T_{lm}$$

Solving for UA:

$$UA = \frac{\dot{m}_{cold} C_{p,H_2O} \Delta T_{cold}}{\Delta T_{lm}}$$

Eq. 16. Characterizing the UA of the double pipe heat exchanger using the hot fluid in the energy balance.

Method 3:

The final method, method 3, used empirical correlations for Nusselt number values for flow in a small tube and flow in an annulus. The equations used are listed in the introduction, equations 9 to 14. The inner tube experience laminar flow at flow rates of 12.6 mL/s to 25.2 mL/s water (Re < 2300) and transitional and turbulent flow occurred at higher flow rates tested (Re > 3000). Turbulent flow correlations were used to model the Nusselt number in transitional flow regimes. The annulus exclusively experienced laminar flow (Re < 2300). Values for fluid properties were found for water assuming a film temperature of $20\ ^{\circ}\mathrm{C}$.

Shell and Tube:

For each shell and tube configuration the Number of Transfer Units (NTU) method was used to calculate UA. The NTU of a system is a dimensionless number which can be used to relate the overall heat transfer coefficient to the minimum heat capacity flow rate.

$$NTU = \frac{UA}{C_{min}}$$

Eq. 17. NTU equation.

The NTU method is commonly used for shell and tube heat exchangers as it can account for a heat exchanger's complicated geometric configuration without having to know exact geometric measurements. The NTU method uses heat capacity flow rates, which are the combination of the mass flow rate and the heat capacity of fluid. C_{min} in equation 17 is the smallest combine mass flow rate and heat capacity of the system. Since the heat exchangers in this study used liquid water, with constant heat capacity, the minimum heat capacity flow rate corresponded to the stream with the lowest mass flow rate. C_{min} account for the maximum possible heat transfer that a fluid can handle in a system.

$$C = \dot{m}C_p\ \left(\frac{W}{K}\right)$$

Eq. 18. Heat capacity flow rate.[5]

NTU calculations can be done for complex geometries since they are based on the effectiveness of the system. The effectiveness of a system is a ratio of the actual heat transfer rate divided by the maximum heat transfer rate possible if the system were to have infinite surface area. The equation for maximum heat transfer rate is given from the maximum temperature difference achieved in the heat exchanger and the maximum amount of heat transfer that can be imparted on the fluids in the system, given by C_{min}.

$$q_{max} = C_{min}(T_{H,in} - T_{C,in})$$

Eq. 19. Maximum HT rate.[5]

$$\varepsilon = \frac{q_{measured}}{q_{max}}$$

Eq. 20. Effectiveness for HT.[5]

Using the measured effectiveness of the system, the NTU can be calculated using the following method.

$$C_r = \frac{C_{min}}{C_{max}}$$

$$E = \frac{\frac{2}{\varepsilon} - (1 + Cr)}{(1 + C_r^2)^{\frac{1}{2}}}$$

$$NTU = -(1 + C_r^2)^{-\frac{1}{2}} \ln \frac{E-1}{E+1}$$

Eq. 21. NTU calculated using effectiveness.[5]

By combining equations 17 and 21, UA, the overall heat transfer coefficient times the area may be found.

NTU Method Terminology

$NTU = Number\ of\ Transfer\ Units$
$\varepsilon = Effectiveness$
$C_{min} = Minimum\ overall\ heat\ capacitance\ \left(\frac{W}{K}\right)$
$C_{max} = Maximum\ overall\ heat\ capacitance\ \left(\frac{W}{K}\right)$
$q_{max} = Max\ theoretical\ heat\ transfer\ rate\ (W)$
$T_{H,in} = Temperature\ of\ Hot\ Stream\ at\ Inlet\ (K)$

$T_{C,in} = Temperature\ of\ Hot\ Stream\ at\ Inlet\ (K)$

For all measurements of UA, at least 50 data points were taken after the system had reached steady state. The average and the 95% confidence interval for UA over those 50+ points were taken and are tabulated in the results section. Small confidence intervals relative to UA values mean that the system had reached steady state when the data was being collected, n. Method 3 for the double pipe heat exchanger does not have confidence intervals since it is based on empirical correlations. This analysis was performed in python. Confidence intervals were calculated from standard error values generated by numpy functions and Z-scores for the 95% confidence interval were calculated using scipy functions.

All ANOVA effect testing was performed in JMP.

Results

Co Current Double Pipe Heat Exchanger

Flow Rates (mL/s)		UA (W/K)				
Cold Flow Rate	Hot Flow Rate	Method 1	Confidence	Method 2	Confidence	Method 3
12.6	12.6	24.07	0.06	20.87	0.07	12.90
12.6	25.2	20.97	0.07	21.54	0.00	14.25
12.6	37.8	22.11	0.23	28.55	0.18	18.04
12.6	50.4	23.79	0.06	32.92	0.07	19.35
12.6	63	16.63	0.73	43.35	0.08	20.15
25.2	12.6	53.18	0.30	65.12	0.39	13.65
25.2	25.2	22.03	0.07	34.02	0.15	15.18
25.2	37.8	25.33	0.19	37.04	0.06	19.55
25.2	50.4	27.00	0.39	42.57	0.06	21.11
25.2	63	28.40	0.14	40.23	0.05	22.06
37.9	12.6	36.10	0.23	49.48	0.24	14.15
37.9	25.2	24.79	0.26	38.29	0.10	15.80
37.9	37.8	29.57	0.29	41.24	0.19	20.59
37.9	50.4	31.57	0.51	40.12	0.17	22.33
37.9	63	31.93	0.09	52.84	0.47	23.39
50.5	12.6	31.88	0.05	59.88	0.41	14.53
50.5	25.2	26.38	0.11	50.03	0.25	16.26
50.5	37.8	31.31	0.05	56.04	0.15	21.39
50.5	50.4	34.04	0.19	54.03	0.08	23.28
50.5	63	33.13	0.34	62.47	0.09	24.43

Table 1. UA Values and 95% confidence intervals for double pipe heat exchanger in a co-current configuration.

A two factor ANOVA was performed on methods 1, 2, and 3 neglecting hot water flow rates of 12.6 mL/s because it is believed that flooding occurred at the temperature sensors for those tests. The analysis examined main effects for hot and cold water flow rate and interactions between cold and hot water flow rates. For methods 1 and 2, both hot and cold flow rates appeared to have an effect, but interactions do not. The analysis done on method 3 was used to demonstrate if the model for UA could return significant effects for varying hot and cold flow rates. As expected, hot and water main effects were observed. The P-Values of these tests are listed in table 2. The low confidence intervals suggest that the system had reached steady state.

Effect	Method 1	Method 2	Method 3
Hot Water	<0.0001	<0.0001	0.0300
Cold Water	0.0168	0.0001	0.0116
Interactions	0.0370	0.0436	0.8265

Table 2. Summary of P-Value of effect tests for double pipe heat exchanger in co-current configuration.

Counter Current Double Pipe Heat Exchanger

Flow Rates (mL/s)		UA (W/K)				
Cold Flow Rate	Hot Flow Rate	Method 1	Confidence	Method 2	Confidence	Method 3
12.6	12.6	111.71	0.22	47.90	0.07	12.90
12.6	25.2	17.41	0.08	2.62	0.00	14.25
12.6	37.8	22.84	0.06	20.06	0.18	18.04
12.6	50.4	23.33	0.10	31.16	0.07	19.35
12.6	63	24.74	0.06	33.42	0.08	20.15
25.2	12.6	53.89	0.39	76.03	0.39	13.65
25.2	25.2	20.74	0.13	32.32	0.15	15.18
25.2	37.8	25.27	0.09	36.36	0.06	19.55
25.2	50.4	28.37	0.18	37.12	0.06	21.11
25.2	63	29.08	0.11	35.71	0.05	22.06
37.9	12.6	79.09	0.53	90.32	0.24	14.15
37.9	25.2	19.03	0.54	41.92	0.10	15.80
37.9	37.8	29.17	0.15	40.04	0.19	20.59
37.9	50.4	30.39	0.24	44.70	0.17	22.33
37.9	63	32.05	0.14	49.26	0.47	23.39
50.5	12.6	79.17	0.49	96.24	0.41	14.53
50.5	25.2	21.46	0.08	45.24	0.25	16.26
50.5	37.8	29.41	0.05	47.53	0.15	21.39
50.5	50.4	32.52	0.09	52.09	0.08	23.28
50.5	63	33.66	0.05	51.61	0.09	24.43

Table 3. UA Values and 95% confidence intervals for double pipe heat exchanger in a counter-current configuration.

The same ANOVA analysis was carried out on each test once again on the counter-current configuration. For methods 1 and 2 significant effects were observed for hot and cold flow rates. Interaction between hot and cold flow rates were observed for method 2. A summary of the results is listed in table 4. The low confidence intervals suggest that the system had reached steady state.

Effect	Method 1	Method 2	Method 3
Hot Water	0.0003	<0.0001	0.0006
Cold Water	<0.0001	0.0029	<0.0001
Interactions	0.1460	0.0436	0.2821

Table 4. Summary of P-Value of effect tests for double pipe heat exchanger in counter-current configuration.

Shell and Tube Heat Exchangers

Flow Rates (mL/s)		UA (W/K)					
Cold Flow Rate	Hot Flow Rate	Co-Current	Confidence	Counter-Current	Confidence	Multipass	Confidence
12.6	12.6	2.95	0.01	9.92	0.03	24.51	0.09
12.6	25.2	46.97	0.18	56.97	0.16	79.46	0.17
12.6	37.8	66.14	0.18	72.94	0.25	88.42	0.37
12.6	50.4	80.95	0.17	77.14	0.26	127.19	0.57
12.6	63	81.41	0.09	73.78	0.42	120.41	0.45
25.2	12.6	6.09	0.06	24.25	0.13	46.10	0.16
25.2	25.2	32.10	0.06	33.61	0.11	40.34	0.21
25.2	37.8	51.15	0.17	54.14	0.09	77.10	0.16
25.2	50.4	65.92	0.39	70.57	0.25	98.81	0.57
25.2	63	76.20	0.23	78.96	0.20	117.96	0.26
37.9	12.6	6.98	0.07	26.91	0.13	48.09	0.20
37.9	25.2	47.23	0.33	55.67	0.32	64.59	0.37
37.9	37.8	38.30	0.16	38.35	0.19	59.22	0.14
37.9	50.4	58.47	0.24	58.48	0.34	91.36	0.38
37.9	63	71.96	0.12	72.87	0.27	111.45	0.28
50.5	12.6	6.39	0.10	19.68	0.26	50.89	0.38
50.5	25.2	58.64	0.11	66.29	0.15	80.98	0.28
50.5	37.8	55.98	0.06	57.07	0.08	82.37	0.16
50.5	50.4	51.34	0.15	51.64	0.10	75.67	0.19
50.5	63	65.58	0.05	66.42	0.17	100.58	0.15

Table 5. UA Values and 95% confidence intervals for shell and tube heat exchangers.

A two factor ANOVA was performed on each heat exchanger testing for main effects of hot water flow rate and cold water flow rate with interactions of hot and cold water flow rate. P-values from effect tests show that hot water flow rate has a significant main effect on all three shell and tube heat exchangers, while cold water flow rate and hot and cold water interactions do not. The low confidence intervals suggest that the system had reached steady state.

Effect	Co-Current	Counter-Current	Multipass
Hot Water	0.0007	0.0052	<0.0001
Cold Water	0.5776	0.826	0.4056
Interactions	0.5633	0.5887	0.1244

Table 6. Results of effect tests on hot and cold water flow rate main effects and interactions for shell and tube heat exchangers.

Discussion

General Considerations for the Heat Exchanger System

Before delving into the effects of flow rate and heat exchanger type on UA, some key observations must be noted.

Flooding

It was noted that the hot water being fed into the heat exchanger systems was filled with air bubbles. At high hot water flow rates, the air bubbles cleared out of the system easily, but at low hot water flow rates the air bubbles could accumulate and combine to form a layer of air within various parts of the heat exchanger systems. This can be referred to as flooding. Flooding was problematic for data measurements, since the temperature sensors (TMP Chips) and the rate of heat transfer within a system could be dramatically affected.

The temperature sensors may read erroneous temperatures as they are not in contact with hot water, but are instead in contact with air that may not be the same temperature as the surrounding hot water. This issue could be observed in the double pipe heat exchanger as it greatly affected the calculation of UA. In general, UA values calculated using method 1, which used the heat transferred from the hot stream, were higher than those calculated using method 3, the expected values from empirical correlations – roughly 1.5 times higher. However, for low hot water flow rates, the UA values calculated using method 1 compared to method 3 could be up to 9 times higher. Method 1 will overestimate UA if heat transfer from the hot water is overestimated. The heat transfer of water is proportional to the difference in temperature between the inlet and outlet temperatures of each stream. It is possible that flooding occurred at the hot water outlet, which lowered the outlet temperature read by the sensor, thus increasing the calculated heat transfer from the hot water. This would lead an overestimate of UA for low flow rates that could not clear out air to prevent flooding. This was observed in every test at low flow rates of 12.6 mL/s for the counter current configuration and one test for the co-current configuration of the double pipe. This implied that there is less flooding at the inlet temperature sensor than the outlet temperature sensor. Though more testing would be needed to validate this assumption, the different orientations of the inlet and outlet sensors suggests one sensor was prone to more flooding issues than the other. The outlet temperature sensor was positioned in a downward orientation while the inlet temperature sensor was positioned in an upward orientation. Since air rises in water due to buoyant forces, in using a downward orientation for a temperature sensor it is likely that a layer of air will form from air bubbling upwards through a slow fluid, hence the resultant flooding. However, in an upward orientation for a temperature sensor, air must move downwards to cover the sensor, and overcome the buoyant forces, a process which is not energetically favorable at low flow rates. Most likely both temperature sensors experienced flooding, but the outlet temperature sensor, in a downward configuration, is believed to have experienced worse flooding, and therefore gave less accurate readings than the inlet temperature sensor, in an upward configuration.

The shell and tube configurations also faced issues as the result of flooding, especially for the single pass heat exchangers. In the single pass shell and tube configurations, the flow of hot fluid was split into 56 separate tubes, each with a fraction of the original flow rate. The layer of air that is formed during the flooding may act as a layer of insulation, limiting the overall heat transfer that can occur within the system. This leads to an overall heat transfer coefficient that is reflective of issues with flooding in the system, more so than the actual heat transfer that occurred within the system. Note that since the flow for the multipass heat exchanger was only split into 14 separate tubes, the multipass heat exchanger was less effected than the single pass heat exchangers by the issue of flooding.

Overall Heat Loss/Gain

The overall heat loss of the system could be characterized using equation 1, and subtracting the heat gained by the cold fluid from the heat lost by the hot fluid.

$$q_{system} = q_{hot} - q_{cold}$$
$$q_{system} = \dot{m}_{hot} Cp_{water} \Delta T_{hot} - \dot{m}_{cold} Cp_{water} \Delta T_{cold}$$

Eq 22. Heat loss from the system.

The heat loss for counter-current double pipe heat exchanger and each shell and tube configuration were calculated for different flow rates using equation 22. The results of these calculations are shown in figure 4.

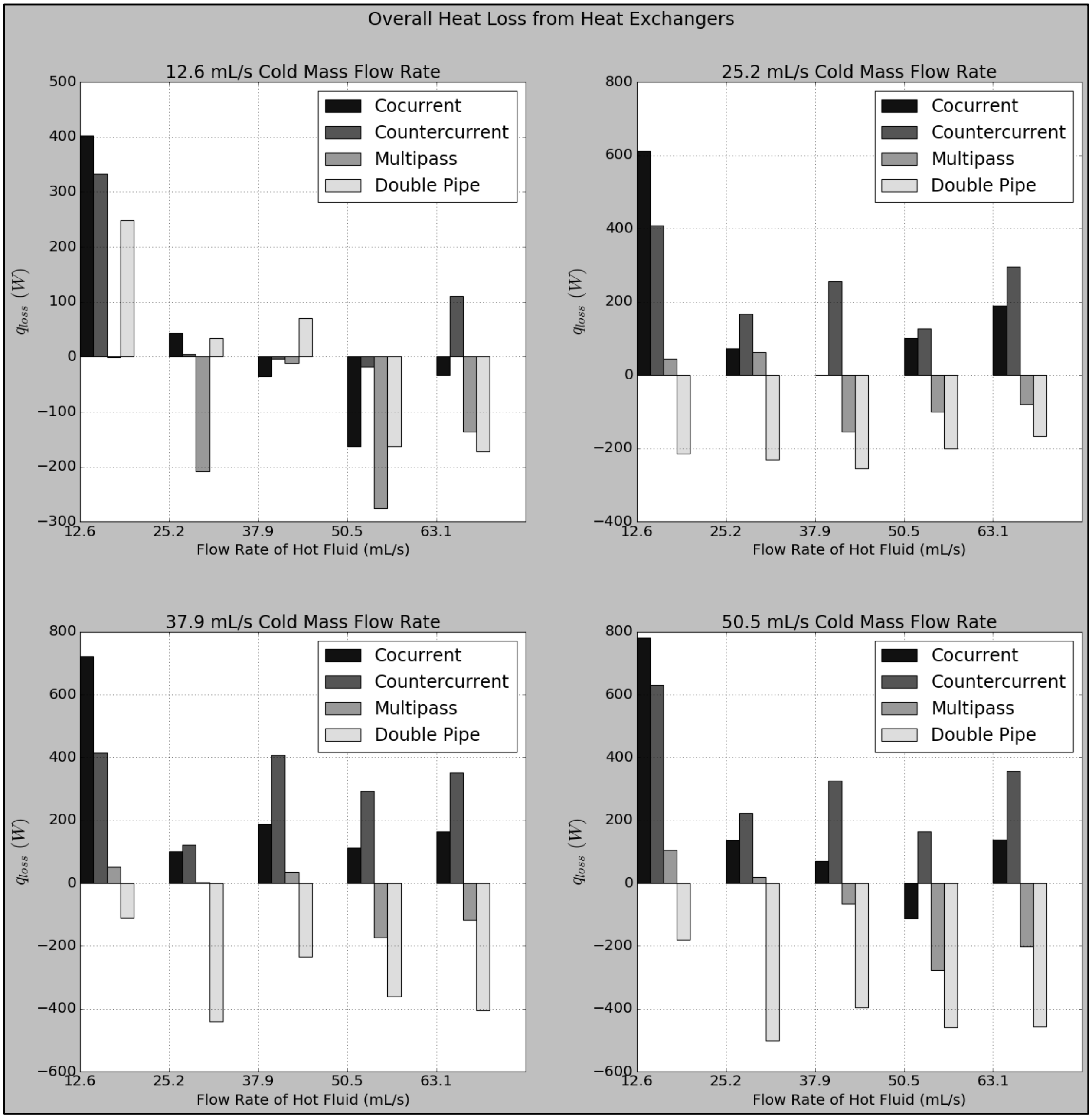

Figure 4. Heat Loss at Multiple Flow Rates.

At low flow rates of hot water, it was observed that the calculated heat loss was highest for the single pass shell and tube heat exchangers. It is observed for the single pass shell and tube heat exchangers that at low flow rates of hot water, heat transfer occurs more readily between the hot water stream and the body of the heat exchanger than the hot water and cold water stream. This could be due to flooding which occurred within the hot water stream, limiting the heat transfer coefficient between the hot and cold water. Since flooding was not as prevalent an issue in the multipass heat exchanger, it did not incur as much heat loss as other shell and tube heat exchange systems at low hot water flow rates.

It was observed that heat could be lost or gained in each system. However, for most flow rates tested, the double pipe heat exchanger had a negative loss of energy. That is, the double pipe heat exchanger often gained more energy than it lost. This is most likely because the cold water stream within the annulus only had a small copper pipe separating the stream from ambient room temperature air. The highly conductive copper pipe allowed significant amounts of heat transfer from the ambient air to the cold water stream. In contrast, it can be observed that overall the two single pass shell and tube configurations released heat into the ambient air for the most part, although the cold stream is on the outside in the single pass configurations as well. This is most likely due to the thick brass shell utilized for these heat exchangers, which has a thermal conductivity approximately three times less than that of copper. The smaller conductivity of brass along with the thickness of the shell allowed the shell and tube heat exchangers to gain less heat from the environment. The multipass heat exchanger had the lowest magnitude of heat loss for any of the heat exchangers. This is most likely because heat transfer between the hot and the cold stream was made favorable by the efficient physical configuration of the multipass system.

The heat loss in the double pipe system to be correlated to cold and hot water flow rates using a two factor ANOVA test. This analysis was performed for the counter-current double pipe heat exchanger, since there was a high amount of heat exchange with ambient are for the double pipe configuration. Main effects were observed for the cold flow rate on heat loss in the system (P-Value < 0.0001); as cold water flow rate increased, the heat gained by the system increased. No effects were observed for hot water flow rate or the interaction between hot and cold water flow rates. It is likely that as the cold flow rate increased, the overall heat transfer coefficient on the outside pipe increased and the temperature gradient between the ambient air and the cold stream increase, leading to higher magnitudes of heat gained by the cold stream.

The heat loss in each system should have effected all calculations, since every method used was derived for adiabatic systems. While there appears to be a similar magnitude of heat lost or gained from the system the heat gain was prominent in the double pipe heat exchanger, which is why three methods were used to calculate overall heat transfer coefficient times area, UA.

Comparison of Methods 1, 2 and 3 for Calculating UA in the Double Pipe Heat Exchangers

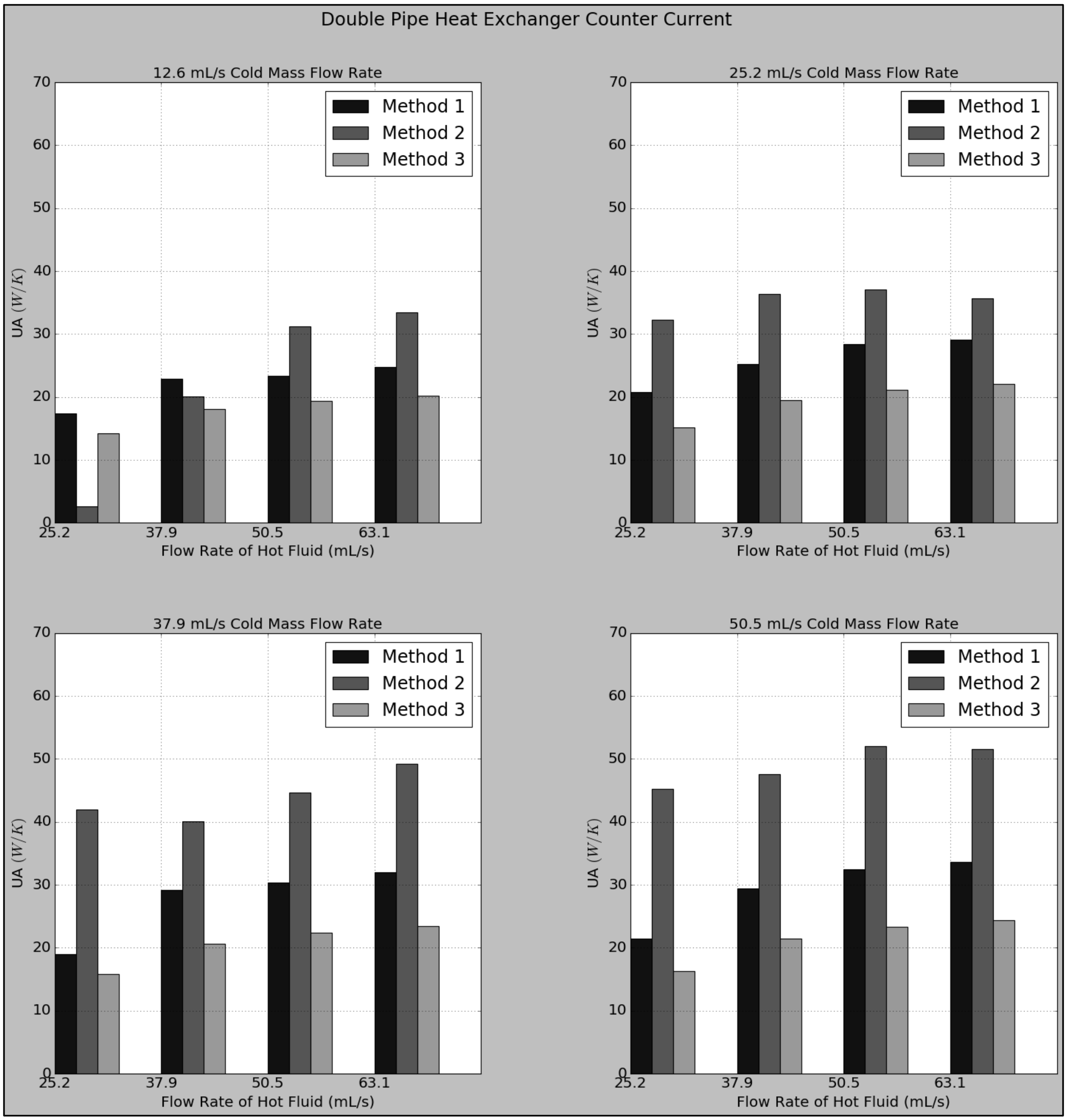

Figure 5. UA values calculated using method 1 (energy balance from hot fluid), method 2 (energy balance from cold fluid), and method 3 (emperical correlations) for counter-current double pipe heat exchanger.

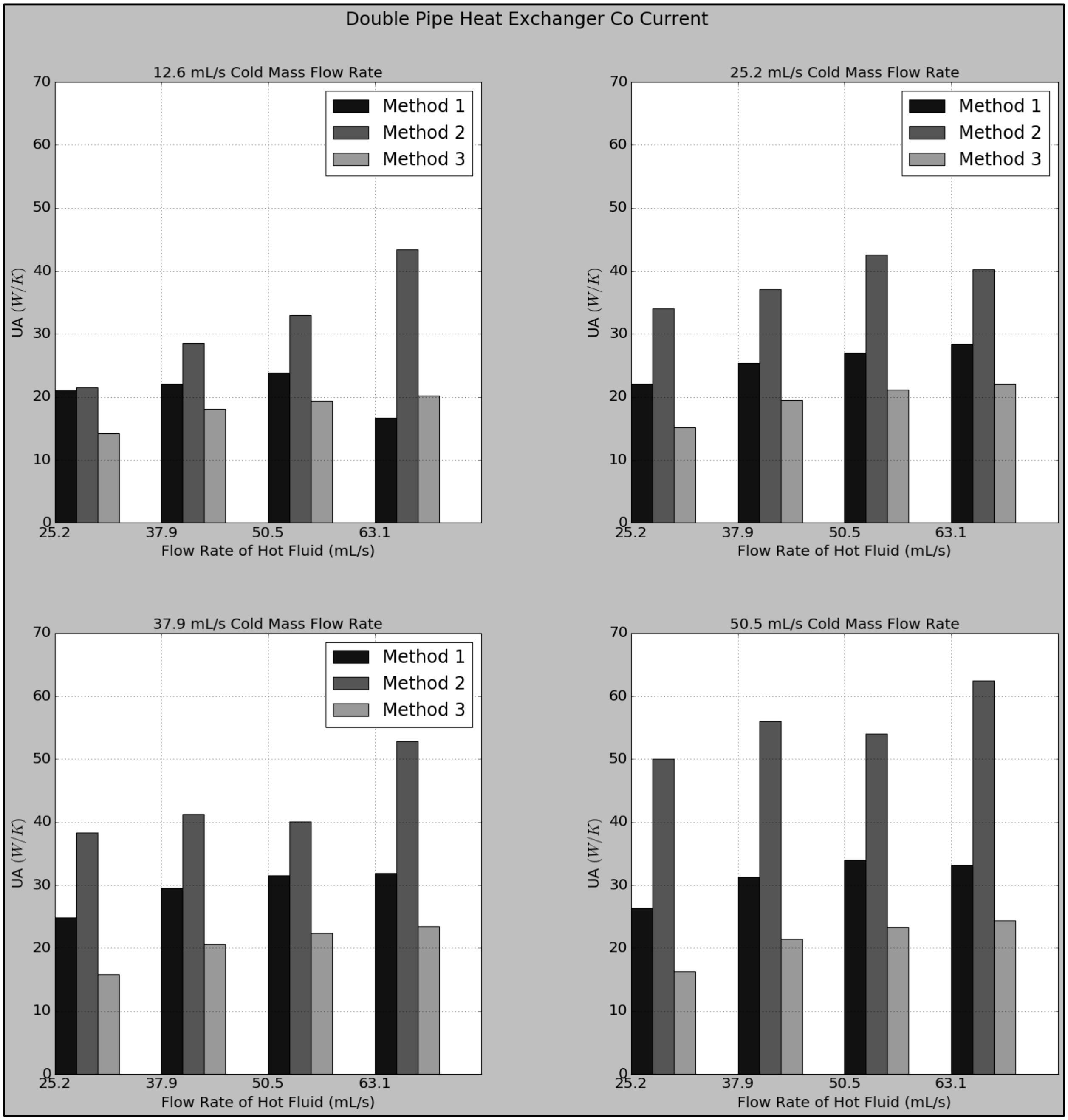

Figure 6. UA values calculated using method 1 (energy balance from hot fluid), method 2 (energy balance from cold fluid), and method 3 (emperical correlations) for counter-current double pipe heat exchanger.

Method 3 is an idealized way to predict the heat transfer coefficient between the hot and the cold streams. Methods 1 and 2 differ in that they were based on experimental data for a non-adiabatic system, meaning that there is a possibility for heat transfer into the system to occur. The heat transferred into the system greatly effects the calculations for UA in methods 1 and 2 and effects the distribution of method 2 at different flow rates. Heat transfer into the system will simultaneously lower the log-mean temperature difference, raise the rate of heat transfer to the cold fluid, and lower the rate of heat transferred from the hot fluid. In method 1, the overall heat transfer coefficient is overestimated for all cases when compared to method 3 due to heat flux into the system. The heat flux into the system has a great effect on the logarithmic mean temperature difference, ΔT_{lm}. In method 1 UA is inversely proportional to ΔT_{lm}; and therefore calculated UA will be overestimated compared to method 3 (see equation 15). A similar effect is observed for method 2, where heat flux into the system increases the temperature difference between inlet and outlet of the cold stream. This increases ΔT_{cold} and decreases ΔT_{lm}, leading to an overestimation of UA (see equation 16).

Method 2 is greatly affected by the heat gain of the system to a far greater degree than method 1. To identify if heat loss had any effect on the calculated UA, a one-way ANOVA test was performed on the data from methods 1 and 2 with respect to the counter-current double pipe heat exchanger. For method 1, no conclusive effect could be justified (P-Value = 0.0814). For method 2 however, there was a strong correlation between heat gain and UA (P-Value ≤ 0.0001). As the heat gain increased, the value of UA increased. This explains why the values of UA calculated through method 2 are nearly double that of methods 1 and 3 at high cold water flow rates, since at those flow rates there is a high amount of heat gained by the system. ‘

Method 1, by visual inspection, seems to be a better representation of the heat transfer coefficient given by correlations in method 3. It is important to note however, that both methods 1 and 2 represent the double pipe heat exchanger in its current uninsulated state. Methods 1 and 3 may be better for predicting what the system will look like when insulated, but the UA values from methods 1 and 2 may both be valid representations of the non-adiabatic system.

While it can be noted that interaction effects between hot and cold water flow rates were present, what caused them could not be determined.

Flow Rate Effects in the Shell and Tube Heat Exchangers

Main effects of hot water flow rate were observed for every heat exchanger examined. Effects of hot water flow rate for the double pipe heat exchanger configurations can be observed in figures 5 and 6 for co-current and counter-current flow, respectively. Effects of hot water flow rate on shell and tube configurations are shown in figure 7. In general, UA increased as hot water flow rate increased. This is expected, as the increased hot water flow rate increases the Reynolds number of the stream, thus decreasing the individual resistance to heat transfer in the stream.

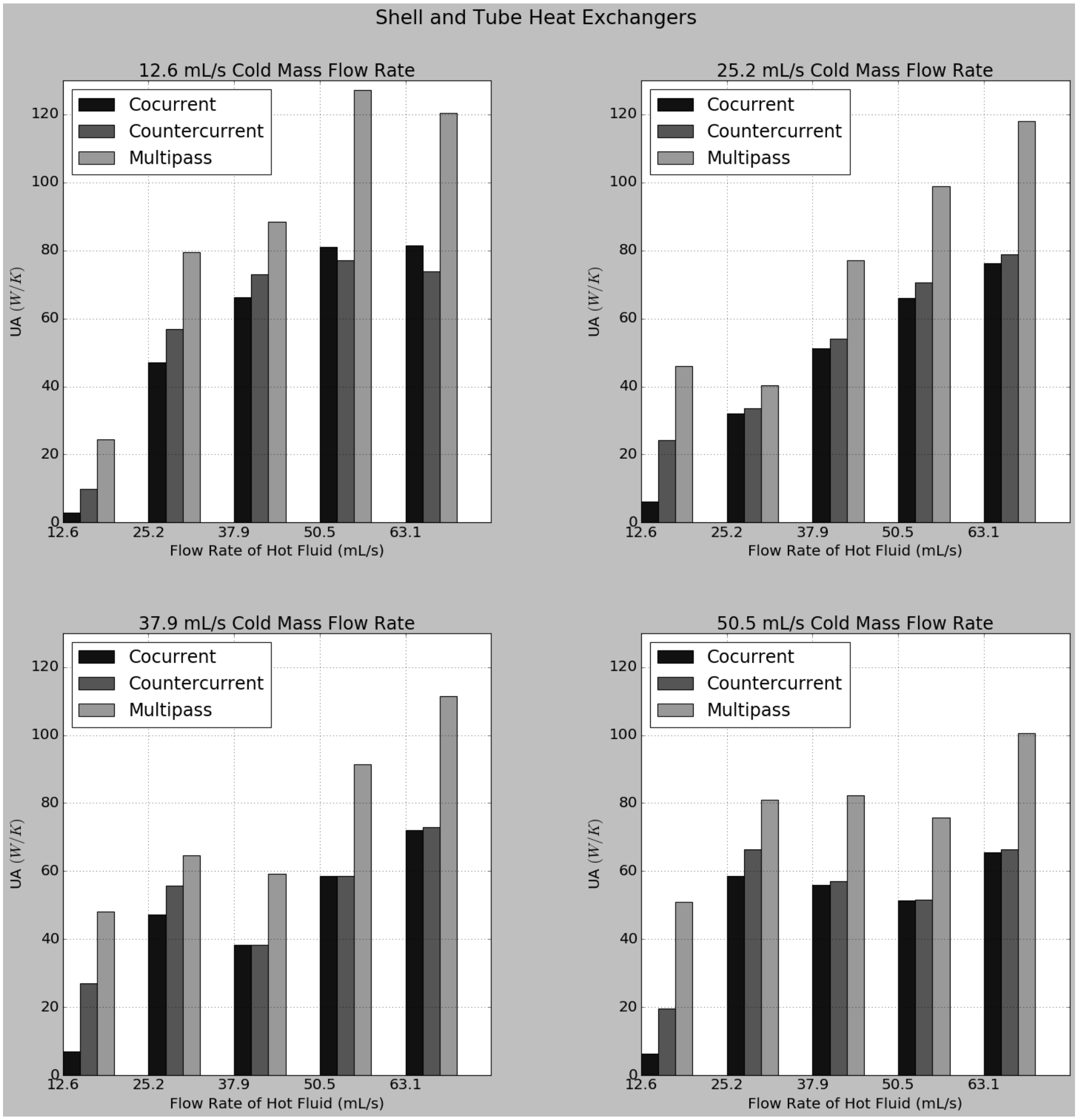

Figure 7. Effects of hot and cold water flow rate on UA for different shell and tube configurations.

For shell and tube heat exchangers, changing the hot water flow rate had a significant effect on UA, but changing the cold water flow rate did not. This means that at the flow rates examined, the hot water flow is a limiting resistance whereas cold water flow is not a limiting resistance in each heat exchanger. If one's goal is to maximize UA efficiently, it is more beneficial to increase the flow rate of hot water than it is to increase the flow rate of cold water for the shell and tube systems.

Heat Exchanger Type and Configuration Effects on Temperature Change

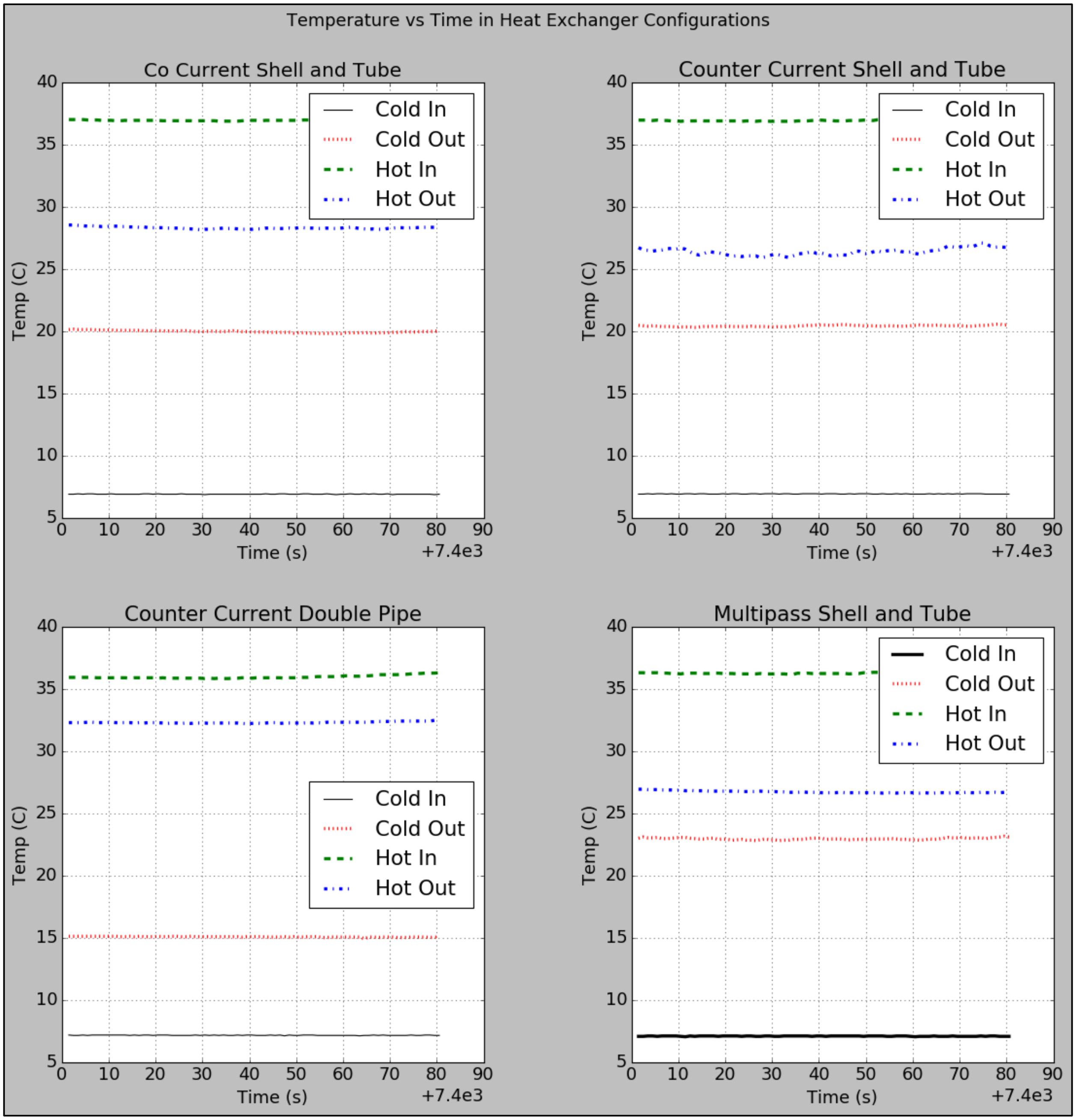

Figure 8. Temperature Readings at cold flow rate of 25.2 mL/s and hot flow rate of 37.9 mL/s.

Co-Current vs Counter-Current Flow

Co-current and counter-current flow returned similar values for UA in both the shell and tube and the double pipe heat exchangers. While there was no effect on UA, in general, the temperature of the hot stream leaving the heat exchanger for counter-current flow was lower than that of hot stream leaving the co-current heat exchanger. This is shown in figure 8 for the co-current and counter-current single pass shell and tube heat exchangers. The calculated UA values for each configuration of heat exchanger were similar, so the difference can only be due to the flow configuration.

When designing a heat exchanger, to maximize the change in temperature in the system it is beneficial to utilize counter-current flow. The enhanced efficiency counter-current flow comes from a more consistent temperature difference between hot and cold fluid down the length of the heat exchanger. A smaller temperature difference between hot and cold fluid leads to a less irreversible heat transfer. If a counter-current heat exchanger is designed properly, the temperature difference between the hot and cold fluid can be treated as negligible, leading to an almost reversible heat transfer between hot and cold fluid.[3]

Single Pass Shell and Tube vs Multipass Shell and Tube

Multipass heat exchangers compared to their single pass counterparts offer higher UA at the same volumetric flow rates for all cases tested. This could be due to the higher amount of flow through each tube, since the hot water flow is only split into 19 tubes rather than 56, leading to smaller resistances in the hot water. For shell and tube heat exchangers of the same size, multipass heat exchangers offer the best heat transfer in terms of UA for all the shell and tube heat exchangers.

Double Pipe vs Shell and Tube

The double pipe heat exchanger used in the tests offered overall worse heat exchange than the shell and tube heat exchangers. As shown by figure 8, the temperature changes in the hot and cold stream achieved by the double pipe heat exchanger are far less than what was achieved in the shell and tube heat exchangers. It is possible to achieve the same amount of heat transfer with the double pipe heat exchanger as the shell and tube heat exchangers by increasing the length of the double pipe heat exchanger

Conclusion

Using multiple method, the overall heat transfer coefficient times the heat transfer area, UA, could be characterized for the double pipe heat exchanger. Method 1 was useful in that it reflected the modeled heat transfer coefficient calculated using method 3, while method 2 was effected by the heat gains of the double pipe heat exchanger. Both methods 1 and 2 are useful in that they can be used to model heat exchange in the non-adiabatic room temperature double pipe heat exchanger.

UA was modeled for each shell and tube heat exchanger using the NTU method, giving similar values for the co-current and counter-current heat exchangers and giving the highest values for the multipass heat exchanger.

Cold and hot water flow rates had effects on heat transfer in the double pipe heat exchanger, increasing cold water flow rate increase the amount of heat gained and increased the heat transfer coefficient from the cold stream. By increasing hot water flow rate, the heat transfer coefficient from the hot stream could be increased.

For the shell and tube configurations, increasing hot water flow rate, rather than cold water flow rate had the largest effect on the heat transfer coefficient. This is because the limiting resistance in the shell and tube heat exchangers was the resistance of the hot water stream.

The multipass heat exchanger was able to achieve the largest temperature change for the hot and cold streams between the inlet and outlet of system. The counter-current heat exchanger was able to achieve a larger temperature change than the co-current heat exchanger due to more efficient heat transfer. The double pipe heat exchangers were able to achieve the lowest temperature change of all the systems.

When designing a heat exchanger system, the cost of materials must be considered in addition to the UA values. While the double pipe heat exchanger yielded the lowest UA values for our system (using methods 1 and 3), the area of heat transfer could easily be increased by increasing the length of the tubing, to achieve the same UA values that can be achieved in the shell and tube heat exchangers at a much lower cost. The drawback of this is that the length of the tubing would need to be around 3 to 4 times its current length, meaning that the heat exchanger would need 3 to 4 m of space. If space is an important factor for one's purchasing decisions, it would be best to go with the multipass heat exchanger, which has the largest UA (and depending on where one buys it may cost the single pass heat exchangers).

Additional Error Analysis

The main source of error in each test was flooding, since at low flow rates the temperature sensor could not properly read the temperature of the water. This is a random error rather than a systematic error, since the amount of bubbles accumulated in the hot stream could vary with time.

Another source of error was in the water provided to the system. The flow rate and the temperature of the hot water provided by the building's water heater varied with time and usage. This could be controlled for by making sure that the temperature data utilized to calculate heat transfer coefficients was taken at steady state. By calculating the confidence interval for the UA values, this demonstrated that the system was at or nearly at steady state, since the confidence intervals for UA were small for every heat exchanger (shown on table 1, 3, and 5). It is important to note that this confidence interval is only used to prove that the system is at steady state, with nearly constant temperatures and volumetric flow rates. It is not a reflection of the error in temperature measurement due to flooding, since that could not be accounted for by the system reaching steady state.

Fouling is a source of thermal resistance that may occur in all heat exchangers, due to the buildup of solids on the walls of the heat exchanger. Fouling is difficult to characterize, and could increase the overall heat transfer coefficient of the heat exchangers. In the characterization of heat exchangers, fouling contributed to the calculations used for methods 1 and 2 for the double pipe heat exchangers and the NTU calculations for the shell and tube heat exchangers. This means that the calculations performed to characterize UA may have been slightly lower than the UA theoretically possible for a clean system. Nevertheless, they were a good representations of the heat exchangers at the time they were measured.

Brief Cost Analysis

It is important to understand how heat exchangers perform with respect to their cost, so that when buying heat exchangers, you make sure you get the best exchanger for your purposes at the lowest cost. In this lab, the shell-in-tube exchangers studied all costed \$369.00 each. Using prices found online, the copper tubing required for the double pipe exchanger would be around \$19.52. It is important to keep the much lower price of the single pipe exchanger in mind when looking at overall heat transfer.[7]

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Appendix A: Terminology

$\Delta T_i = Temperature\ Difference\ Between\ Hot\ and\ Cold\ Stream\ at\ Inlet\ (K)$

$\Delta T_o = Temperature\ Difference\ Between\ Hot\ and\ Cold\ Stream\ at\ Outlet\ (K)$

$UA = \ Overall\ Heat\ Transfer\ Coefficient\ times\ Area \left(\frac{W}{K}\right)$

$\dot{m}_{hot} = Mass\ Flow\ Rate\ of\ the\ Hot\ Stream\ \left(\frac{g}{s}\right)$

$\dot{m}_{cold} = Mass\ Flow\ Rate\ of\ the\ \text{Cold}\ Stream\ \left(\frac{g}{s}\right)$

$C_{p,H_2O} = Heat\ Capacity\ of\ Water\ = 4.18\frac{J}{gK}$

$\Delta T_{hot} = Temperature\ Difference\ Between\ Inlet\ and\ Outlet\ of\ Hot\ Stream\ (K)$

$\Delta T_{cold} = Temperature\ Difference\ Between\ Inlet\ and\ Outlet\ of\ Cold\ Stream\ (K)$

$\Delta T_{lm} = Logarithmic\ Mean\ Temperature\ Difference\ (K)$

$Nu_D = Nusselt\ Number\ (average)$

$h = Individual\ HT\ Coefficient\ \left(\frac{W}{m^2K}\right)$

$k_f = Fluid\ Conductive\ HT\ Coefficient\ \left(\frac{W}{mK}\right) = .597\frac{W}{mK}\ at\ 293\ K\ for\ water$

$Re_D = Reynold's\ Number\ (average)$
$u = Velocity\ of\ Stream\ \left(\frac{m}{s}\right)$
$D_e = Hydraulic\ Diameter\ (m)$

$\Delta T_{cold} = Temperature\ Difference\ Between\ Inlet\ and\ Outlet\ of\ Cold\ Stream\ (K)$
$h_i = Individual\ Heat\ Transfer\ Coefficient\ for\ Pipe\ \left(\frac{W}{m^2K}\right)$
$h_a = Individual\ Heat\ Transfer\ Coefficient\ for\ Annulus\ \left(\frac{W}{m^2K}\right)$
$L_p = Measured\ length\ of\ pipe\ (m)$
$D_{i,i} = Measured\ Inner\ Diameter\ of\ Inner\ Tube\ (m)$
$D_{i,o} = Measured\ Inner\ Diameter\ of\ Outer\ Tube\ (m)$

$D_{o,i} = Measured\ Outer\ Diameter\ of\ Inner\ Tube\ (m)$

$\mathrm{Pr} = \text{Prandtl Number}$

$NTU = Number\ of\ Transfer\ Units$
$\varepsilon = Effectiveness$
$C_{min} = Minimum\ overall\ heat\ capacitance\ \left(\frac{W}{K}\right)$

$C_{max} = Maximum\ overall\ heat\ capacitance\ \left(\frac{W}{K}\right)$
$q_{max} = Max\ theoretical\ heat\ transfer\ rate\ (W)$
$T_{H,in} = Temperature\ of\ Hot\ Stream\ at\ Inlet\ (K)$
$T_{C,in} = Temperature\ of\ Hot\ Stream\ at\ Inlet\ (K)$

Appendix B: Sample Calculations For Selected Equations

Equation 5

$$Re = \frac{vL_c}{\upsilon} = \frac{.2327556\ {}^{m}/_{s} * \ .01439\ m}{.995 * 10^{-6}\ {}^{m^2}/_{s}} = 3366.184$$

$$Ra_D = GrPr = \left(1.327 * 10^8 \frac{1}{K * m^3} * (14.5838K) * (.02846m)^3\right)(.708) = 31584.90447$$

$$Nu_{free} = \left(.6 + \frac{.387 Ra_D{}^{\frac{1}{6}}}{(1 + (.559/Pr)^{\frac{9}{16}})^{\frac{8}{27}}}\right)^2 = \left(.6 + \frac{.387(31563.348)^{\frac{1}{6}}}{(1 + (.559/.708)^{\frac{9}{16}})^{\frac{8}{27}}}\right)^2 = 5.787$$

$$Gz = RePr\frac{L_c}{L_p} = (1141.9)(6.96) * \frac{.02660m - .01582m}{.762m} = 112.43489$$

Equation 11

$$Nu_{Outer\ Laminar} = 1.2\left(3.66 + \frac{.0668Gz}{1 + (.04Gz)^{\frac{2}{3}}}\right) = 1.2\left(3.66 + \frac{.0668(112.43489)}{1 + (.04(112.43489))^{\frac{2}{3}}}\right)$$
$$= 6.81178$$

$$f = (1.58\ln(Re_D) - 3.28)^{-2} = (1.58\ln(3366.184) - 3.28)^{-2} = .01096$$

Equation 10

$$Nu_{Inner\ Turbulent} = \frac{\left(\frac{f}{2}\right)(Re_D - 1000)Pr}{1 + 12.7\left(\frac{f}{2}\right)^{\frac{1}{2}}\left(Pr^{\frac{2}{3}} - 1\right)} = \frac{\left(\frac{.1096}{2}\right)(Re_D - 1000) * 6.96}{1 + 12.7\left(\frac{.1096}{2}\right)^{\frac{1}{2}}\left((6.96)^{\frac{2}{3}} - 1\right)}$$
$$= 25.8812$$

Equation 9

$$Nu_{Inner\ Laminar} = 1.86\left(\frac{d_i RePr}{L_p}\right)^{1/3} = 1.86\left(\frac{(.01439m)(1122.06)(6.96)}{.762m}\right)^{1/3} = 9.8271$$

Equation 8

$$Nu = \frac{hL_p}{k} \quad \rightarrow \quad h = \frac{Nu * k}{L_p} = \frac{25.8812 * .597\ {}^{W}/_{Km}}{.762m} = 1073.74\ \frac{W}{m^2K}$$

Equations 12, 13, and 14

$$U_o = \frac{1/A_o}{\frac{1}{h_i A_i} + \frac{\ln(d_o/d_i)}{2\pi k L_p} + \frac{1}{h_o A_o}}$$

$$= \frac{\frac{1}{.075743m^2}}{\frac{1}{\left(407.6964\ {}^{W}/_{m^2K}\right)(.068896m^2)} + \frac{\ln\left(\frac{.01582m}{.01439m}\right)}{2\pi\left(386\ {}^{W}/_{Km}\right)(.762m)} + \frac{1}{\left(315.2593\ {}^{W}/_{m^2K}\right)(.075743m^2)}}$$

$$= 12.898\ \frac{W}{m^2K}$$

Appendix C: Physical Properties for water at 293 K

$$\nu = Kinematic\ Viscosity\ \left(\frac{m^2}{s}\right) = 0.995 * 10^{-6}\ \frac{m^2}{s}$$

$$C_{p,H_2O} = Heat\ Capacity\ of\ Water\ = 4.18\frac{J}{gK}$$

$$\mathrm{Pr} = \mathrm{Prandtl\ Number} = 6.96$$

Appendix D: Material Properties

Thermal Conductivity of Copper at 298 K $= k_{copper} = 386\frac{W}{mK}$

Thermal Conductivity of Brass at 298 K $= k_{brass} = 107\frac{W}{mK}$